CS109 Section 1 Solutions

Problem 1

Let C be the event that the student knows geometry. Let S be the event that the student solves the first geometry question correctly. Then $P(C) = q$, $P(C^c) = 1 - q$, $P(S|C) = p_1$, and $P(S|C^c) = p_2$. The value we need to solve for is $P(C|S)$. We use Bayes Rule:

$$P(C|S) = \frac{P(S|C)P(C)}{P(S)}$$
$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|C^c)P(C^c)}$$
$$P(C|S) = \frac{p_1 q}{p_1 q + p_2(1-q)}$$

Problem 2

Let S be the event the student solves the first question correctly.
A = the event the student is in the Beginner level
B = the event the student is in the Intermediate level
C = the event the student is in the Expert level
D = the event the student is in the Euclid level
Then $P(A) = b_1$, $P(B) = b_2$, $P(C) = b_3$, $P(D) = b_4$. And $P(S|A) = d_1$, $P(S|B) = d_2$, $P(S|C) = d_3$, $P(S|D) = d_4$. We want to solve for $P(A|S)$, $P(B|S)$, $P(C|S)$, $P(D|S)$. Here we show the example for $P(A|S)$. We use Bayes Rule:

$$P(A|S) = \frac{P(S|A)P(A)}{P(S)}$$
$$P(A|S) = \frac{P(S|A)P(A)}{P(SA) + P(SB) + P(SC) + P(SD)}$$
$$P(A|S) = \frac{P(S|A)P(A)}{P(S|A)P(A) + P(S|B)P(B) + P(S|C)P(C) + P(S|D)P(D)}$$
$$P(A|S) = \frac{d_1 b_1}{d_1 b_1 + d_2 b_2 + d_3 b_3 + d_4 b_4}$$

Note that we get the second equation by the law of total probability.

Problem 3

Let event E = email is actually non-spam. Let event F = email is marked as GOOD by the filter. We compute q as follows:

$$q = P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}$$

Note that $P(E) = p$, and $P(F|E) = 1$ since an email that is non-spam will always be marked GOOD by the filter (whether we encounter the bug or not). We also know that $P(F|E^c) = 0.1$, since a spam email is only marked as GOOD if we encounter the bug (which happens 10% of the time). Thus, we have:

$$q = \frac{p}{p + (0.1)(1-p)} = \frac{p}{0.1 + 0.9p}$$